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13. ABSTRACT (Maximum 200 words)

This research developed both the theory and algorithms capable of providing realistic control systems for physical plants of interest to the Air Force which are modeled as distributed parameter systems. Much of the focus has been on developing algorithms to obtain the inner-outer factorization (IOF). Having the factorization allows one to stablize systems, design compensators, and find, in general, control strategies to infinite-dimensional systems. We have established important relationships between the IOF, well-posedness, and the mixed sensitivity problem. Control solutions have been extended to a wider class of single-input single-output infinite dimensional systems, as well as a specific class of multi-input multi-output systems. The IOF has also been extended to a specific class of irrational H^∞ matrices.

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"Control Theory for Distributed Parameter Systems"

David S. Flamm, original PI

Stuart C. Schwartz, PI

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(a) Objectives:

The basic objective of this research effort has been to develop both theory and algorithms capable of providing realistic control systems for physical plants of interest to the Air Force which are modeled as distributed parameter systems, i.e., infinite dimensional linear systems. Applications include large space structures and precision pointing of optical and radar systems.

(b) Accomplishments and Progress: Executive Summary

In this section , we provide a summary of the progress and accomplishments under this project. More detailed discussions are included as Appendix 1.

In developing feedback control systems for infinite dimensional linear systems, the major stumbling block has been to perform the inner-outer factorization (IOF): one must compute an all-pass (inner) factor and a minimum phase (outer) factor in order to solve the control problem.

We have analyzed the IOF and have developed numerical algorithms to accomplish the factorization and have studied the relationship to the "well posedness" of the control problem. We have also been able to relate the well-posedness to the mixed sensitivity problem: well-posedness implies that the solution to the mixed sensitivity problem is also well posed. Consequently, we now have a clear connection between the IOF and solving the H^∞ mixed sensitivity problem. Conversely, these results help to characterize when a H^∞ problem is ill-posed, i.e., when, for example, an all-pass system is approximated by a minimum phase factor.

The criterion we have developed is a norm bound between the system and ones knowledge of the system. Let $\{f_n\} \subset H^\infty$ be the sequence of approximations and $f \subset H_\infty$ the true system. Then, if

$$\int_{-\infty}^{\infty} |\frac{\log |f_n| - \log |f|}{1 + t^2}| dt \longrightarrow 0$$

the problem is well posed, and the sequence of outer factorizations converges to the outer factor of the true system. In addition, if we also have uniform convergence, $f_n \longrightarrow f$, we also get the inner factorization of f_n to converge to f.

The relationship developed between IOF and well posedness also contributes to the mixed sensitivity problem: well-posedness implies the solution to the mixed sensitivity problem is also well posed. Additional results relate to developing stabilizing compensators and the interpolation of compensators. (See Appendix 1.)

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We have also extended the solution of single-input single-output infinite dimensional systems to hold for a wider class of plants, and have also made extensions for a specific class of multi-input multi-output problems. In addition, we have been able to obtain the IOF for a specific class of irrational H^∞ matrices. Further details are given in Appendix 1.

(c) Written Publications

1. D. S. Flamm, "Outer Factor 'Absorption' for H^∞ Control Problems," *Int. J. Robust and Nonlinear Control*, Vol. 2, No. 1, May 1992, pp. 31-48.

2. D. S. Flamm and H. Yang, "Optimal Mixed Sensitivity for SISO Distributed Plants," to appear in *IEEE Trans. Automatic Control*

3. H. Yang and D. S. Flamm, "Mixed Sensitivity Design for a Class of Multivariable Infinite Dimensional Systems," (submitted to *IEEE Trans. Automatic Control*).

4. D. S. Flamm and K. M. Crow, "Application of Numerical Inner-Outer Factorization to Computation of H^∞-Optimal Performance," to appear in *Proc. 32nd IEEE Conf. Decision and Control*, San Antonio, TX, Dec. 15-17, 1993.

5. K. M. Crow and D. S. Flamm, "Analysis and Approximation of Ideal Compensators for Distributed Parameter SISO H^∞ Mixed Sensitivity Problems," to appear in *Proc. 1993 Conf. Information Sciences and Systems*, Baltimore, MD, Mar. 1993.

6. H. Yang, "H^∞ Optimal Compensators for a Class of Infinite Dimensional Systems," *Proc. American Control Conf.*, San Francisco, CA, June 2-4, 1993, pp. 2170-2174.

7. H. Yang and D. S. Flamm, "A Lower Bound for H^∞-Optimal Performance," (with H. Yang), *Proc. 31st IEEE Conf. Decision and Control*, Tucson, AZ, Dec. 16-18, 1992, pp. 2259-2260.

8. H. Yang and J. M. Orszag, "Numerical Computation of H^∞ Optimal Performance, *J. Scientific Computing*, Dec. 1992.

9. D. S. Flamm and K. K. Crow, "Numerical Computation and Well-Posedness of H^∞ Optimality Criteria for Feedback Control Systems," *Proc. 31st IEEE Conf. Decision and Control*, Tucson, AZ, Dec. 16-18, 1992, pp. 2268-2273.

10. H. Yang and D. S. Flamm, "Mixed Sensitivity Design for a Real Multivariable Delay Problem," *Proc. American Control Conf.*, June 26-28, 1991, Boston, MA, pp. 2202-2207.

11. D.S. Flamm and K. Klipec, "Numerical Methods for H^∞ Control of Distributed Parameter Systems," in *Analysis and Optimization of Systems: State and Frequency*

Domain Approaches for Infinite Dimensional Systems (*Proc. 10th Int. Conf.*, Sophia-Antipolis, France, 9-12 June 1992), Lecture Notes in Control and Information Sciences **185**, Springer-Verlag, 1993, pp. 598-609.

12. D. S. Flamm, H. Yang, Q. Ren and K. Klipec, "Numerical Computation of Inner Factors for Distributed Parameter Systems," . 29th Allerton Conf. Communications, Control and Computing, Oct. 2-4, 1991, pp. 72-73.

13. D. S. Flamm and K. M. Crow, "Numerical Computation of H^∞-Optimal Performance for Feedback Control Systems and Well-Posedness of Optimality Criteria," *in preparation.*

14. "H^∞-Optimal Mixed Sensitivity for General Distributed Plants", in *Proc. IEEE Conf. on Decision and Control*, December, 1990.

15. "H^∞-Optimal control for a class of infinite dimensional MIMO systems", in *Proc. of Conference on Information Sciences and Systems*, March, 1991.

(d) Professional Personnel

Two graduate students have been associated with the project, resulting in two Ph.D. theses.

1. H. Yang, "H^∞ Control for Infinite Dimensional Systems," June 1993

2. K.M. Crow, "Algorithms for H^∞ Control of Distributed Parameter Systems," January 1994

(e) Interactions

Please see (c), Written Publications.

Appendix 1A

Research Summary: Katherine M. Crow

This research examines the practical implementation of direct solution of H^∞ optimal control problems for distributed parameter systems. We have outlined a procedure for numerical computation of the optimal performance and optimal compensator starting from a general plant which may be irrational and unstable. In fact, this procedure can be applied to a plant described only by numerical frequency response data. In addition, we have specified conditions under which the H^∞ problem is well-posed in terms of the continuity of the performance criterion under small perturbations of the problem data. These results relate to the numerical conditioning of the computational procedure, and therefore are of direct practical significance. In order to motivate this work, we have been using several distributed parameter transfer function models as test cases. Specifically, we have been working with transfer functions for an Euler-Bernoulli beam with two kinds of damping, an Euler-Bernoulli beam with a tip mass, and a Timoshenko beam, in addition to a delay with one stable pole added.

In order to exactly solve the H^∞ optimal control problem, one must find the inner-outer factorization of an H^∞ function. For the finite dimensional case, if one knows the poles and zeros of the transfer function, the factorization may be done by inspection; otherwise state space algorithms are available. The latter approach is not feasible for irrational transfer functions, and only a very few simple examples may be factored by inspection. Therefore, we have developed an algorithm to perform this factorization numerically.

To justify this algorithm, we have proven two kinds of convergence results. The first gives sufficient conditions for the inner factor of an approximant to converge to the inner factor of the true function, whether the factorization is done numerically or exactly. This result is important since it describes one set of conditions for how well a model can describe the actual system in order for the H^∞ solution to make sense. The second describes high frequency behavior of the transfer function which is sufficient to truncate the infinite range of integration in the numerical procedure. This result is also vital in describing the assumptions which are adequate when one's knowledge of the system transfer function consists of measured data and thus necessarily only contains information about a finite frequency range. Finally, we have proven that the H^∞ performance criterion is continuous in the inner factor: that is, if a sequence of plant models satisfies the assumed convergence conditions, the H^∞ performance criterion will converge to that for the true system. In addition to validating our computational techniques, these results answer the question of well-posedness of the H^∞ control problem, providing conditions under which the H^∞ solution is continuous for plant approximations.

For unstable plants, a coprime factorization must be found in order to compute the optimal performance. In the rational case, state space representations are used with an observer-based compensator to formulate a coprime factorization. However, for irrational transfer functions, state space computation is not an option. Instead, feedback stabilization may be used to determine a coprime factorization for the plant from a coprime factorization for a particular stabilizing compensator. This method may be applied experimentally when a transfer function for the plant is not explicitly known to arrive at a numerical coprime factorization; it may also be used to obtain a symbolic coprime factorization when the transfer function model is known. Analysis of the effect of numerical error in this procedure

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has also been performed.

Depending upon the problem formulation, the optimal compensator may be improper, which poses problems for implementation. We have described a method for approximating the improper optimal compensator with a proper one and have proven that one can choose an approximation with performance arbitrarily close to optimal. These approximation techniques have been demonstrated on a model with an irrational outer factor, which could not be treated by previous methods.

In addition, the numerical inner-outer factorization is a pointwise computation, and pointwise knowledge of the inner factor leads to pointwise knowledge of the optimal compensator. We would like to have a symbolic representation of the compensator for implementation purposes; hence we must interpolate the compensator. Here we use "interpolate" to mean computing an approximation from pointwise information about the true function. We have computed bounds on the permissible error in such an approximated compensator to describe its stability and performance properties. These bounds provide justification that if we can approximate with enough accuracy (by increasing the number of points at which we compute the optimal compensator), we can guarantee the performance and stability of the resulting compensator.

Appendix 1B

Research Summary: Hong Yang

1 Results

We summarize in the following the main results obtained while the author was supported by the grants mentioned in the title.

1. We observe that for a general class of SISO infinite dimensional linear time invariant systems, including unstable systems, the resulting Hankel plus Toeplitz type operator is a finite rank perturbation of the Hankel + Toeplitz type operator studied by Zames and Mitter. We show that this finite rank perturbation operator can be computed explicitly. We also fill in some missing points that are required for actual computation in the Zames-Mitter work.

2. For a class of MIMO infinite dimensional linear time invariant systems, we give new characterizations for the eigenspaces of the corresponding Hankel + Toeplitz type operator. We use these new characterizations to prove that the eigenspaces are finite dimensional for this class of Hankel

   + Toeplitz type operator. Moreover, we show how to explicitly compute the eigenvalues and eigenvectors. We believe that this is the first time that a class of true multiple input-single output infinite dimensional linear time invariant systems is considered for the H^∞ optimal mixed sensitivity design. We also show how to reduce the original Hankel + Toeplitz type operator so that its essential spectrum can be computed explicitly.

3. For computing the H^∞ optimal mixed sensitivity in the nonunique case, we give new characterizations for certain subspaces of some Kreĭn spaces. The new characterizations are used to compute the H^∞ optimal mixed sensitivity in some cases for which it could not be computed before.

4. Using the characterizations mentioned in 3 above, we give some numerical schemes for computing the approximations of H^∞ optimal performance and H^∞ optimal pure and mixed sensitivity for a general class of SISO systems. Although we have not considered the convergence of the algorithms, the algorithms have been tested on some infinite dimensional systems and proved to be very efficient. A major advantage of the algorithms is that the complexity does not depend on the dimensionalities of the systems and weights. The algorithms are easy to implement.

5. We show how to compute the inner-outer factorization for a class of irrational H^∞ matrices. These factorizations are needed for the H^∞ optimal mixed sensitivity design.

The results hence represent significant progress not only towards the theoretical understanding of infinite dimensional H^∞ optimization problems but also towards the numerical solution of complex engineering problems. We note that our approach is highly technical and mathematical. However, it is also practical, as we show with numerical examples.